New Jersey

Mathematics

Curriculum Framework

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(with contributions from many other New Jersey educators)

December 1996

A Collaborative Effort of the

New Jersey Mathematics Coalition

and the

New Jersey Department of Education

PREFACE

The *New Jersey Mathematics Curriculum Framework* is based on the *Mathematics Standards* adopted by the New Jersey State Board of Education on May 1, 1996. The *Mathematics Standards* are part of the *Core Curriculum Content Standards,* developed by the New Jersey State Department of Education, including standards in seven content areas and cross-content workplace readiness standards. Taken together, the *Core Curriculum Content Standards* describe what every New Jersey student needs to understand and be able to do at the completion of the 4th, 8th, and 12th grade.

PURPOSE: New Jersey's *Mathematics Standards* describe a vision of mathematics teaching and learning which involves high expectations for *all* students, and insists that *all* students can achieve these expectations.

The *New Jersey Mathematics Curriculum Framework* provides information and guidance to teachers and districts on how to help make that vision a reality.

Achieving this vision is an ambitious, long-term undertaking; there is no simple path to the goal. Achieving this vision will take time, effort, and a commitment to change. The recommendations of the *Mathematics Standards* cannot be implemented overnight, and results will not appear overnight. Changes will be needed in all areas — in curriculum, instruction, assessment — and will involve rethinking school practices and extensive professional development. The changes will require the commitment of teachers, administrators, school boards, parents, and policy makers, and the effort of the entire New Jersey community.

The *New Jersey Mathematics Curriculum Framework* is intended to be a resource, providing practical guidance to implement the *Mathematics Standards*. It includes information and resources for teachers at all grade levels and for school and district administrative personnel. Each chapter contains much information, and can serve as a basis for extended discussions involving teachers and administrators.

The *New Jersey Mathematics Curriculum Framework* is not intended to be read straight through. It is intended to be user-friendly; but to achieve that purpose, the user also has to be friendly, warming up to its contents a little at a time, and not shying away from it because of its bulk.

New Jersey's *Mathematics Standards* and the *New Jersey Mathematics Curriculum Framework* are directed toward one crucial goal:

GOAL: To enable all of New Jersey's children to move into the twenty-first century with the mathematical skills, understandings, and attitudes that they will need to be successful in their careers and daily lives.

The *Mathematics Standards* are based on the twin premises that all students can learn mathematics and that all students need to learn mathematics. They set high achievable expectations for all students, and call for teachers and parents to help all students strive toward and achieve those standards.

New Jersey's *Mathematics Standards* and the *New Jersey Mathematics Curriculum Framework* call for

major changes, both in terms of what mathematics will be taught, and in how it will be taught. The recommendations provided here are very specific. Yet, it is not intended that they be implemented dogmatically; different situations call for different responses and different strategies. In education, as in other areas, there is a tendency to swing from one extreme to another. We hope that educators will utilize their common sense, judgment, and experience in finding appropriate ways of using the recommendations in this *Framework* to inform their decision-making. We expect that this *Framework* will be a major resource to teachers seeking to implement the *Mathematics Standards* in the classroom; we also expect it to be valuable to districts which are seeking to introduce mathematics curricula based on the *Mathematics Standards* and to provide professional development to their teachers based on the *Mathematics Standards*.

The publication of this document is the culmination of the New Jersey Mathematics Curriculum Framework Project, a collaborative effort of the New Jersey Mathematics Coalition and the New Jersey Department of Education, which was funded by an Eisenhower grant from the United States Department of Education. This effort is also a component of New Jersey's Statewide Systemic Initiative to Improve Mathematics, Science, and Technology Education. The *Framework* and the *Mathematics Standards* build on the Standards published by the National Council of Teachers of Mathematics in 1989 and 1991.

A *Preliminary Version* of the *New Jersey Mathematics Curriculum Framework* was published in January 1995. That *Preliminary Version* reflected the efforts of hundreds of New Jersey mathematics educators who worked together during 1993 and 1994 to develop materials that would be appropriate for a world-class mathematics curriculum framework. During the last two years, the *Preliminary Version* has been reviewed and used by many teachers, schools, and districts throughout the state. This new version of the *New Jersey Mathematics Curriculum Framework* reflects their comments and suggestions, and follows the organization of the *Mathematics Standards* adopted by the New Jersey State Board of Education.

Though published, the *New Jersey Mathematics Curriculum Framework* is not completed. We anticipate that it will continue as a living document on the Web site of the New Jersey Mathematics Coalition, where it is available at http://dimacs.rutgers.edu/nj_math_coalition/framework.html/. We hope to post additional resources relating to the *Mathematics Standards*, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

The efforts of all those who have contributed to the development of the *New Jersey Mathematics Curriculum Framework* are acknowledged below. This has been truly a state-wide effort of which we can all be proud. Let us all continue to work together to make the vision of New Jersey's *Mathematics Standards* a reality in the coming years!

For further information, please call the New Jersey Mathematics Coalition at 908/445-2894 or contact the New Jersey State Department of Education, Office of Standards and Assessment, CN 500, Trenton, NJ 08625-0500. We welcome your comments on the *Framework* and your suggestions about its future; please send them to joer@dimacs.rutgers.edu or the New Jersey Mathematics Coalition, P.O. Box 10867, New Brunswick, NJ 08906.

Joseph G. Rosenstein
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Co-Director, New Jersey Mathematics Curriculum Framework Project
Professor of Mathematics, Rutgers University
Director, New Jersey Mathematics Coalition
December 9, 1996

ACKNOWLEDGEMENTS

The development of the *Preliminary Version* of the *New Jersey Mathematics Curriculum Framework* was a broad-based effort. As evidence, I submit the following section (pages v-xi) which appeared in the *Preliminary Version* acknowledging all those who had roles in the development of that version of the *Framework*. They should be pleased to see that all their efforts have now come to fruition. Many thanks to all of those who played a role in developing the *Framework*!

By contrast, this version was the result of an intense effort by a small number of people. Warren D. Crown, Professor of Mathematics Education at Rutgers, Janet H. Caldwell, Professor of Mathematics at Rowan College of New Jersey, and Joseph G. Rosenstein, Professor of Mathematics at Rutgers University wrote this entire document, based on the materials in the *Preliminary Version* and responding to the comments and suggestions offered by all those who used and reviewed that document.

Assisting in the writing process was Karin Rupp who collected and organized all of the comments and suggestions, and developed additional information that grew out of those comments and suggestions. Also assisting in the writing process were those who carefully read various chapters of the *Preliminary Version*, and recommended many changes. This includes Robert Davis, Frank Gardella, Evan Maletsky, and Maureen Quirk. Especially important was the contribution of those who reviewed the chapter dealing with Discrete Mathematics, a topic which has never before been subjected to a K-12 grade-level analysis; this includes Valerie DeBellis, Emily Dann, Bobbie Goldman, Janice Kowalczyk, Evan Maletsky, Claire Passantino, and Michael Saks, as well as the many teachers in the Leadership Program in Discrete Mathematics who shared their classroom experiences with these topics.

Before this version of the *New Jersey Mathematics Curriculum Framework* could be written, the *Mathematics Standards* had to be adopted by the New Jersey State Board of Education. So acknowledgements are appropriate here to those who served on the Governor's Review Panel for the Mathematics Curriculum Standards — Janet Amenhauser, Joyce Baynes, Janet Caldwell, Warren Crown, Barbara Graham, Patricia Klag, Paul Lawrence, Evan Maletsky, Paula Norwood (Panel co-Chair), Jean Paige, Robert Riehs (Department of Education), Joseph G. Rosenstein, William Smith (Panel co-Chair), Dorothy Varygiannes (Department of Education), and Allen Wesley.

The editing of the *New Jersey Mathematics Curriculum Framework* was the work of Joseph G. Rosenstein, with the dedicated assistance of Karin Rupp. Both have read each word of this document ... over and over. Meeting the deadline imposed by the Eisenhower grant period — this document had to be printed by December 31, 1996 — involved, simply put, many long days and nights.

The document was prepared by the staff at the Center for Mathematics, Science, and Technology Education at Rutgers University, including the staff of the New Jersey Mathematics Coalition. Most of the document was put into its final form by Stephanie Micale and Debby Toti who have cheerfully put up with the *Framework*, and its Editor, full-time for the last three months. Chris Magarelli did all the computer graphics, and others provided important assistance when it was needed — Janet DeBellis, Valerie DeBellis, Lisa Estler, Bonnie Katz, Stephanie Lichtman, and Peter Sobel. There were times when six people were working on the document simultaneously! Thank you for all your help.

Thanks also to Dolores Keezer of the Department of Education, who has served as co-Chair of this project and has helped ensure the dissemination of this document, and to Robert Riehs of the Department of

Education who has helped ensure that the contents of this document reflected the initiatives of the Department. Thanks also to Stephen Bouikidis for providing expert advice on the format of this document and to him and Keith Kershner for ensuring that its cover would do justice to its contents. In addition to their assistance, the Mid-Atlantic Eisenhower Consortium for Mathematics and Science Education at Research for Better Schools also provided a graphic artist for the cover and contributed to the distribution of the *Framework*. Thanks also to Rutgers University and the Center for Mathematics, Science, and Computer Education for serving as a home for this project, and to its Director, Gerald A. Goldin, for his consistent support.

Finally, thanks to my family for their patience during the last few months while I was intensely wrapped up in the writing and editing of the *New Jersey Mathematics Curriculum Framework*.

Joseph G. Rosenstein
Editor, New Jersey Mathematics Curriculum Framework
Co-Director, New Jersey Mathematics Curriculum Framework Project
December 9, 1996

ACKNOWLEDGEMENTS

(from *Preliminary Version* — 1995)

The development of the *Preliminary Version* of the *New Jersey Mathematics Curriculum Framework* was made possible by a grant from the United States Department of Education to the New Jersey Department of Education and the New Jersey Mathematics Coalition. Joseph G. Rosenstein has served as co-Director of the New Jersey Mathematics Curriculum Framework Project for the New Jersey Mathematics Coalition; serving as co-Directors for the New Jersey Department of Education have been Charles Mitchel, Karen Sanderson, and Dolores Keezer.

In addition to a collaboration between the New Jersey Mathematics Coalition and the New Jersey Department of Education, this document represents a collaboration involving many of the most knowledgeable mathematics educators in New Jersey and many other members of the community. Some served on the New Jersey Curriculum Standards Panel that developed the draft version of the *New Jersey Mathematics Standards*. Others were active members of the Curriculum Framework Project Advisory Committee of the New Jersey Mathematics Coalition. Still others served as members of Task Forces which developed recommendations and drafted materials for the framework. Lists of members of these groups are provided on the following pages[1]; please bring corrections or omissions to our attention so that modifications can be made in subsequent versions.

We also acknowledge the 294 individuals who submitted comments on the draft version of the *New Jersey Mathematics Standards* and the 29 District Leadership Teams (DLTs) who reviewed an earlier version of the framework as part of their participation in the pilot implementation program of the New Jersey Mathematics Curriculum Framework Project. Serving as Project Coordinator of the pilot implementation program have been Irwin Ozer and Karin Rupp.

Special mention must be made of the following individuals who have served the Leadership Team in various capacities, attending long and arduous planning meetings, chairing Task Forces, and writing, reviewing, and editing endless drafts of sections of this document.

Janet Caldwell, Professor of Mathematics, Rowan College of New Jersey
Warren Crown, Associate Professor of Mathematics Education, Rutgers University
Karl-Heinz Haas, Mathematics and Science Supervisor, Saddle Brook Public Schools
Dolores Keezer, Director, Academy - Central, New Jersey Department of Education
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Elizabeth Marquez, Teacher, North Brunswick Township Schools
Bruce Normandia, Superintendent, Brick Township Public Schools
Anthony Piccolino, Associate Professor of Mathematics, Montclair State University

[1] Members of the New Jersey Mathematics Curriculum Standards Panel are listed on page vii, of the Curriculum Framework Project Advisory Committee on page v, and of the various Task Forces on pages viii-ix. A list of the 60 districts comprising the District Leadership Teams (DLTs) and the DLT Coordinators appears on pages x-xi.

Joseph G. Rosenstein, Professor of Mathematics, Rutgers; Director, NJ Mathematics Coalition
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Larry Wiley, Mathematics Consultant, (formerly) New Jersey Department of Education

Special thanks to Phyllis Klosowski who has served as Project Coordinator of this project since its inception, and has been a source of information and assistance to the Leadership Team and to Mary Ann Reilly, who developed the initial draft of the framework upon which this *Preliminary Version* is based. Thanks also to the Mid-Atlantic Eisenhower Consortium for Mathematics and Science Education located at Research for Better Schools for providing refreshments for those reviewing framework drafts.

That this document now exists is a tribute to a team of five people each of whom has spent hundreds of hours converting an idea into a reality:

Karen Sanderson, currently Assistant Principal, Hamilton (Atlantic County) Public Schools, who served for nearly two years as Project co-Director for the New Jersey Department of Education;

William Smith, Mathematics Supervisor, Haddonfield Public Schools, who served as Facilitator of the *New Jersey Mathematics Standards* Panel;

Janet Caldwell, Professor of Mathematics, Rowan College of New Jersey, and
Warren Crown, Associate Professor of Mathematics Education, Rutgers University, and Associate Director, New Jersey Mathematics Coalition

who were principal authors of the *Preliminary Version* of the *New Jersey Mathematics Curriculum Framework* with Joseph G. Rosenstein, writing, rewriting, and weaving together the contributions of many others; and

Joseph G. Rosenstein, Professor of Mathematics, Rutgers University, and Director, New Jersey Mathematics Coalition

who served as Project co-Director, and headed the team and managed the effort to create this document.

Congratulations and thanks to all who have participated in developing the *Preliminary Version* of the *New Jersey Mathematics Curriculum Framework*.

New Jersey Mathematics Standards Panel

Ray Allen	Atlantic City, Teacher
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Irene Benn	Toms River, Special Education
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(affiliations as of completion of the work of the Panel in June 1993)

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Gloria Sanok	Teacher, Wayne Schools
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Patricia Wang-Iverson	Research for Better Schools
Linnea Weiland	New Jersey Department of Education
Larry Wiley	New Jersey Department of Education
Joseph Zahn	New Jersey Department of Education

District Leadership Teams (DLTs) and DLT Coordinators

New Jersey Mathematics Curriculum Framework Project

Regional Leadership Team I

Atlantic City DLT	Ray Allen
Mainland Regional DLT (including Linwood, Northfield, Somers Point)	Daniel Kortvelesy
Ocean City DLT (including Sea Isle, Upper Township)	Sherry Bosch
Stone Harbor DLT (including Middle Township, Dennis Township, Avalon)	Lynn Mathis, Ronald Bonner, Tom Cushane

Regional Leadership Team II

Haddonfield DLT	William Smith
Bridgeton DLT	Rita Sherman
Pitman DLT	Barbara Michalsky
Salem City DLT (including Quinton)	Esther Lee
Pittsgrove Township DLT (including Elmer)	Carol Taylor Winkie, Jacqueline Murphy

Regional Leadership Team III

Florence Township DLT	Dana Paykos
Northern Burlington County Regional DLT (including Chesterfield, Mansfield, North Hanover, Springfield)	Ellen Mulligan
Trenton DLT	Mary Mitchell
Metuchen DLT	Joseph Bulman
South Brunswick DLT	Kaye L. Crown

(continued)

District Leadership Teams (DLTs) and DLT Coordinators (continued)

New Jersey Mathematics Curriculum Framework Project

Regional Leadership Team IV

Pinelands Regional DLT (including Eagleswood Township, Little Egg Harbor Township, Tuckerton Elementary, Bass River)	Harry Bublin
Southern Regional DLT (including Stafford Township, Long Beach Island Township)	Deborah Mayeux
Eatontown DLT (including Monmouth Regional)	Diane Bloom
Ocean Township DLT	Leona Worth
Rumson-Fairhaven Regional DLT (inlcuding Rumson, Fairhaven)	Louis W. Mitchell

Regional Leadership Team V

Midland Park High School DLT (including Ho-Ho-Kus)	Patricia A. Dunn, Victoria Kniewel
East Orange DLT	Howard Walker, James Leutz
Dover DLT (including Mine Hill)	Gary Meyer
Jefferson Township DLT	Mary Ann Tierney
Sparta Township DLT	Pat Spagnoletti, Karl Mundi

Regional Leadership Team VI

Union City DLT	Silvia Abbato
Washington Township DLT (including West Morris)	Dolly Cinquino
Franklin Township DLT	Jane Taglietta
Bernardsville DLT	Louis Rodriguez
Linden DLT	Walter Tylicki

TABLE OF CONTENTS

INTRODUCTION TO THE *FRAMEWORK*

Overview

The publication of the *New Jersey Mathematics Curriculum Framework* is the culmination of the New Jersey Mathematics Curriculum Framework Project, a four-year collaborative effort of the New Jersey Mathematics Coalition and the New Jersey Department of Education, which was funded by an Eisenhower grant from the United States Department of Education. This effort is also a component of New Jersey's Statewide Systemic Initiative for Improving Mathematics, Science, and Technology Education.

The purpose of the *New Jersey Mathematics Curriculum Framework* is to provide a guide to individual New Jersey teachers, administrators, and districts that will help them translate a vision of exemplary mathematics education into reality. It is anticipated also that the *New Jersey Mathematics Curriculum Framework* will serve as a model for other states.

The *New Jersey Mathematics Curriculum Framework* intends to help educators base their district's mathematics curriculum on the recommendations of New Jersey's *Mathematics Standards*. It illustrates how each of the standards can be addressed at all grade levels, and provides information and guidance on the major issues that need to be addressed, on the process of systemic change, and on the inter-related areas of content, instruction, and assessment.

Defining *Standards* and *Framework*

We may think of *standards* as expressing our common goals — first in terms of a vision, and then in terms of clear statements (called *standards*) of what we want to accomplish. A useful metaphor is that of a road map, where the goal is simply a common destination. You *do* have to know where you're going before you can figure out how you're going to get there. Moreover, since we are all starting at different places, we will take very different routes to arrive at our common goal.

We may think of a *framework* as an instrument to help us determine which route to use, how to structure our efforts, in order to achieve our goal. A useful metaphor is that of a skeletal structure. The framework is not a completed building. It is, however, the scaffolding that provides initial support, definition, and direction to our efforts to achieve our goal.

New Jersey's *Mathematics Standards* are intended to describe our goals; the *New Jersey Mathematics Curriculum Framework* is intended to help us achieve those goals. It is intended to provide policy-makers, instructional leaders, teachers, and community members with the support, definition, and direction necessary to re-envision and reconstruct mathematics education here in New Jersey and across the United States. The *New Jersey Mathematics Curriculum Framework* is not a finished product — it is not a curriculum; it does however provide the support necessary for educators who wish to generate and implement a new vision of how mathematics can be taught and learned in their schools.

... All Students

The vision that is presented in New Jersey's *Mathematics Standards* and the *New Jersey Mathematics Curriculum Framework* is articulated in high standards which are indeed achievable by all New Jersey

students. All students *need* to achieve these standards if they are to be productive in the 21st century; all students *can* achieve these standards if we create environments in which learning is both possible and expected. There may be exceptions, but these must be exceptional.

At the same time, our attention to "all students" must not diminish our dedication to providing full encouragement and opportunity to explore mathematics in greater breadth and depth to those students who have interest or talent in pursuing careers which require additional mathematical achievement.

Standards and Frameworks in a National Context

This document builds on the *Curriculum and Evaluation Standards for School Mathematics* (1989) and the *Professional Standards for Teachers of Mathematics* (1991), published by the National Council of Teachers of Mathematics.

The 1993 report from the National Governors Association to the National Education Goals Panel entitled *Promises to Keep: Creating High Standards for American Students* recommended and announced the development of national standards documents in seven other content areas. A basic theme of both the Goals 2000 legislation (March 1994) and the Improving America's Schools Act (October 1994) is the importance of developing high standards of learning for all students. A national consensus has been building around the importance of agreed-upon standards in improving the education of our country's students.

Standards and Frameworks in a New Jersey Context

A draft version of the *New Jersey Mathematics Standards* was developed by a panel of thirty-one individuals who met extensively during the 1992-1993 school year. Crafted by a broad range of New Jersey elementary, middle school, and secondary teachers, supervisors, administrators, college mathematics educators, mathematicians, and representatives of business and industry, the draft *New Jersey Mathematics Standards* was intended to provide a clear vision of exemplary mathematics learning and to define and then articulate the standards necessary for achieving quality mathematics education.

After the completion of the draft *New Jersey Mathematics Standards*, over 7000 copies of the document were distributed for review across the state. At the same time, efforts began to extend the draft *New Jersey Mathematics Standards* into a mathematics framework. The *Preliminary Version* of the *New Jersey Mathematics Curriculum Framework*, published in January 1995, contained a revised version of the standards which addressed many of the comments of both the reviewers of the draft standards and the drafters of the framework materials. As a result of this process, the standards in the *Preliminary Version* represented a statewide consensus of what mathematics educators believe are high achievable goals for all students.

During 1995, a new working group — the Governors's Review Panel for the Mathematics Curriculum Standards — built upon these draft standards and, together with similar working groups in other content areas, engaged the public in an extensive review process that resulted in modest modifications of the draft standards in mathematics. This process culminated in the adoption on May 1, 1996 by the New Jersey State Board of Education of the *Core Curriculum Content Standards*, which includes the *Mathematics Standards*, standards in six other content areas, and cross-content workplace readiness standards.

The Organization of the *New Jersey Mathematics Curriculum Framework*

The *New Jersey Mathematics Curriculum Framework* begins with a chapter entitled *New Jersey's*

Mathematics Standards. This chapter presents the vision for mathematics education in New Jersey on which the *Mathematics Standards* are based. It presents the standards that articulate that vision, and it enumerates *cumulative progress indicators* that futher define and elaborate on those standards. It describes them in terms of student experiences, providing a number of vignettes that both illustrate the vision and clarify the recommendations of the standards. The *Mathematics Standards* include the sixteen *content standards* that were adopted by the New Jersey State Board of Education, and two *learning environment standards* that were developed and approved by the task forces that prepared the *Mathematics Standards* and appear in the New Jersey State Department of Education's *Core Curriculum Content Standards*; however, since they were not considered content standards, they were not presented to the New Jersey State Board of Education for adoption.

The next chapter *The First Four Standards* discusses the processes of problem solving, reasoning, communicating mathematics, and mathematical connections that should underlie all student learning; among the "connections" discussed are the connections between mathematics and science.

Each subsequent standard has its own chapter. Each of these chapters begins with a K-12 overview of the standard, and continues with grade-level discussions for each of the K-2, 3-4, 5-6, 7-8, and 9-12 grade levels; these grade-level discussions include grade-level overviews of the content standard followed by sample activities — about 1500 altogether — for achieving the expectations enumerated in New Jersey's *Mathematics Standards* at those grade levels.

The *New Jersey Mathematics Curriculum Framework* is designed so that a teacher can easily extract information about all content areas for a particular grade level, and so that a teacher or an administrator can easily extract information about a particular content area for all grade levels.

There are two additional "planning" chapters, *Implementing a Technology Plan*, which provides guidance on strengthening a district's technology component, and *Planning for Change*, which focuses on the process of bringing about change. All of the recommendations in the *Mathematics Standards* involve significant changes in how mathematics will be taught and learned. System-wide changes involve decisions and actions at all levels: at the district level, at the school level, at the department level, and in the classroom. This chapter discusses how change takes place, both in general and in the specific contexts of professional development for school personnel and aligning school and district policies with mathematics education reform. *Planning for Change* will help you understand the change process and function as a "change agent."

Using the *New Jersey Mathematics Curriculum Framework ...*

A long document like this is not written with the expectation that it will be read from cover to cover. However, it is expected that every reader will begin by reviewing this *Introduction* and the chapter *New Jersey's Mathematics Standards* which follows.

Each chapter of the *New Jersey Mathematics Curriculum Framework* can serve as a basis for extended discussions involving teachers and administrators, and readers are encouraged to form groups in their schools and districts for this purpose.

The *New Jersey Mathematics Curriculum Framework* addresses two audiences. First, it speaks to school and district personnel who intend to implement the standards comprehensively and systemically, by bringing about change in all of their classrooms. Second, it addresses teachers who are interested in implementing the standards in their own classrooms. How each of these groups might use this document is discussed in the

next two sections.

... for Systemic Change

For school and district leaders, the first and last chapters of this *Framework*, those dealing with the *Mathematics Standards* and *Planning for Change*, are critical. Chapter 20 provides a model for understanding systemic change, and describes specific processes to follow in order to successfully bring about change. Key to the success of efforts designed to bring about systemic change is enlisting the involvement and support of all those affected by the change.

But what changes should be made? From the outset it must be acknowledged that "implementing the standards" cannot happen overnight, that there is no "magic bullet," that there is no one action which will transform all of our classrooms, all of our teachers, all of administrators, and all of our students, so that they all manifest the vision. Bringing about change involves a long process, with many inter-related components. Each district must choose specific areas with which to begin its efforts.

We suggest that, in addition to this chapter, you also peruse the various other chapters in the *Framework*, together with your colleagues. Try to reach a tentative agreement on which specific areas in these sections should be the focus of your attention. Those chapters can then be the subject of intensive review and discussion, and subsequently the focus of efforts to improve the mathematics curriculum.

It should be noted, however, that the success of such efforts will depend on whether sufficient attention is devoted to the issues raised in Chapter 20, *Planning for Change*. Thus, for example, decisions about where to focus a school's attention should involve all those within the school who will be involved in implementing those decisions.

... for Change in the Classroom

Teachers should begin by reviewing the chapter on the *Mathematics Standards*, and should then review the grade-level sections in each of the subsequent chapters. The information in each of these chapters is organized by grade-level. Thus, for example, a 5th grade teacher can easily review the grade-level 5-6 material for all of the content standards; this will include overviews of each of the content standards for this grade level, cumulative progress indicators regarding student performance at this grade level, as well as activities intended to help achieve the expectations for each of the standards. The 5th grade teacher should, however, also review the grade-level 3-4 material — to find out what the student is expected to bring to the 5th grade — and the grade-level 7-8 material — to find out what the student will be expected to do at the next grade level.

Summary

The *New Jersey Mathematics Curriculum Framework* presents a vision and a working guide to help educators create the changes necessary to achieve world-class mathematics programs in all New Jersey classrooms. This *Framework* is intended to serve as a vehicle for change, to generate commitment, and to encourage and facilitate the leadership necessary to transform mathematics education in the state. As the authors of the report *Everybody Counts* concluded: "The challenges are clear. The choices are before us. It is time to act." So too, we must accept the challenges, recognize the choices, and take action now.

References

Mathematical Sciences Education Board. *Everybody Counts*. Washington, DC: National Academy Press, 1989.

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New Jersey State Department of Education. *New Jersey Mathematics Standards*, draft, 1993.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards.*